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ON THE ESTIMATION OF LONG RANGE SOUND INTENSITIES BY RAY TRACE METHODS

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On the estimation of long range sound

intensities by ray trace methods

C. S. Clay

The problem of calculating ray trace intensities at short range is
relatively simple when a few particular ray paths from the source to the
receiver are known. Our interest here is in the larger ranges, i.e., ranges
much greater than the "skip distances". Diffraction, scattering and the
changes of the velocity profiles over the large range make it convenient,
even a necessity, to regard the ray paths as being distributed over a
region of the ocean.

To avoid argument, ~~we~~ it is agree that the particular ray paths from a
source position to a receiver position, regardless of the ray, can be
calculated for a sound velocity profile over the path. Temporal changes
and lack of <u>exact</u> knowledge of the sound velocity profile make this
deterministic problem a futile exercise. ~~We shall use~~ the ray tracing
procedure, will be used primarily to determine the aperture at the source and the
distribution of the energy in the region of the receiver. Hence, ~~we can
only estimate~~ the average sound intensity, can only be estimated.

On Fig. 1, we have shown several different classes of rays. At
short range, one has the familiar ray diagrams. At large range, the dis-
tribution of the rays produces a field somewhat like that indicated on the
right. The angular aperatures of insonification are $\Delta\theta_{sd}$, $\Delta\theta_{br}$ and $\Delta\theta_{r}$.
For a surface duct, the attenuation limits the range to short distances and
most of its energy eventually goes into leakage arrivals. Their contribution
is generally small. At short range, the hydrophones "a" and "b" may or
may not observe signal as dependent upon the insonification. At large range,
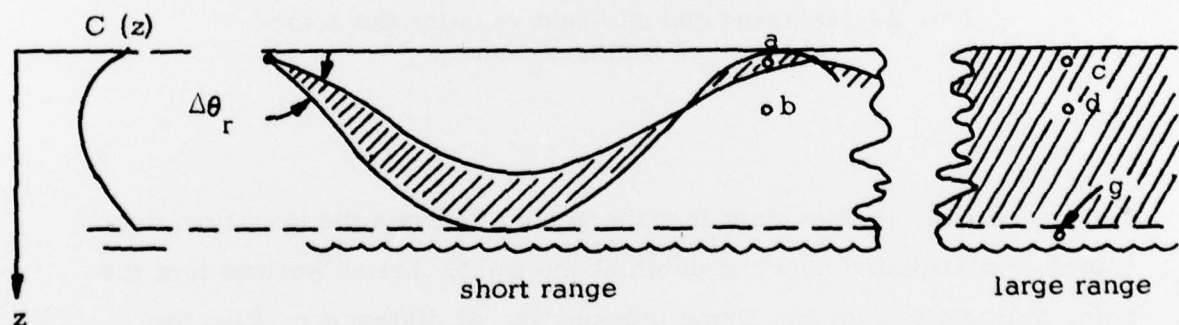the energy is distributed between the upper and lower reflection or turning

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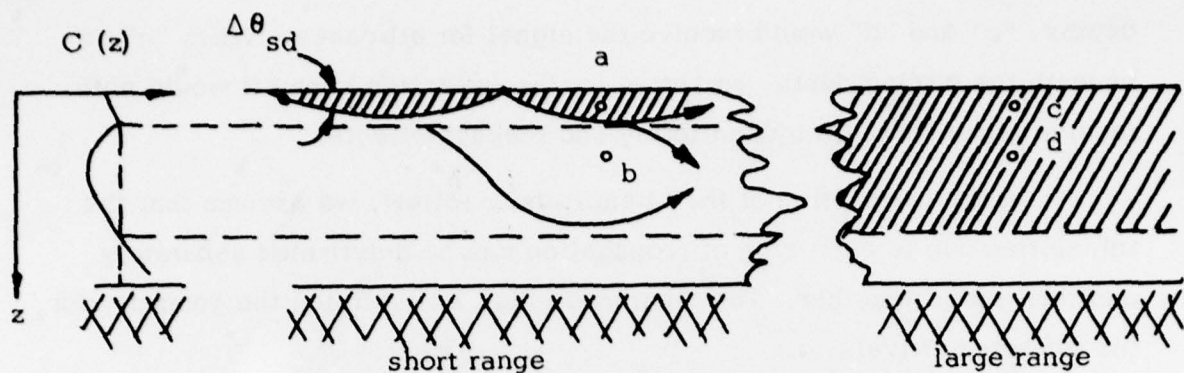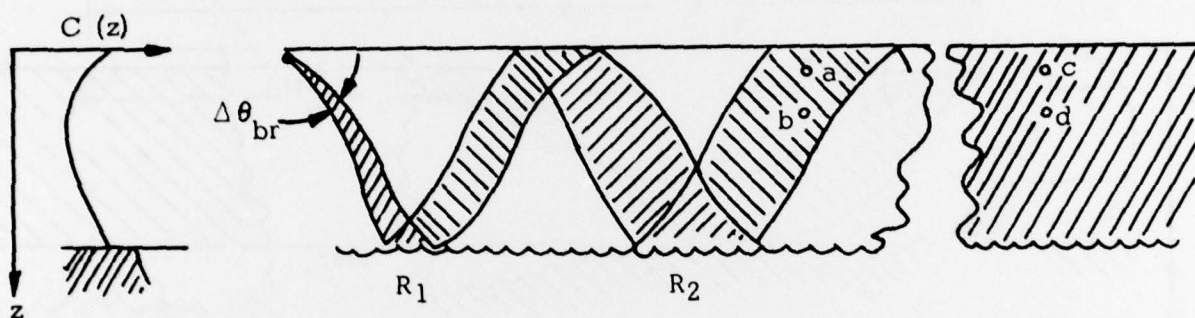
Fig. 1 Types of ray paths. The areas of insonification are shaded and only down going rays are shown.

depths, "c" and "d" would receive the signal for all cases. When "g" is beneath the turning depth, as shown for the refraction case, it would not receive signals except by scattering and leakage effects.

In the calculation of the intensities to follow, we assume that the intensities due to each type of propagation can be determined separately and then added together. The diagram on Fig. 2 illustrates the geometry for the refracted arrivals.
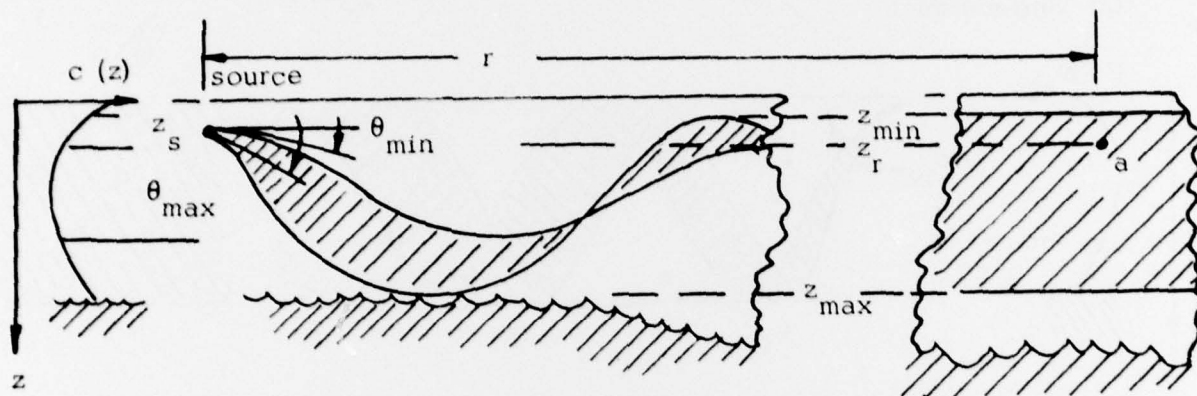
Fig. 2   Maximum and minimum rays for the source
and receiver depths z_s and z_r

From the figure, it is evident that the energy between the rays from the source is distributed over the depth at the right. Let us assume that the point source gives an rms sound pressure p_o at distance r_o from the source. Ignoring the negligible change of $c(z)$ near the source and for small $\Delta\theta$, the outgoing energy is proportional to $2\pi p_o^2 r_o^2 \cos\theta$ $(\theta_{max} - \theta_{min})$. Ignoring, for the present, the absorption and other losses, the energy at distance r is proportional to $2\pi r \int p^2 \, dz$.

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The constant of proportionality is the relation between intensity and $\int p^2 \, dz$ and is nearly the same for both the source and receiver. By conservation of energy, we can write

$$p_o^2 \, r_o^2 \, \cos \theta \, \Delta \theta = r \int_0^h p^2 \, dz \qquad (1)$$

$$\Delta \theta = \theta_{max} - \theta_{min}$$

$$\theta = (\theta_{max} + \theta_{min})/2$$

$$h = \text{depth of the ocean} \qquad (2)$$

where $\Delta \theta$ is in radians and r_o, r, and h are in the same units. Sometimes the range at which the spreading changes from spherical to cylindrical spreading is used to define a quantity equivalent to $\Delta \theta$ (Urick, 1967).

The dependence of p^2 upon depth is an important quantity in the determination of the signal at a receiver. At the beginning, we suggested that if the receiver is above the minimum turning depth or beneath the maximum turning depth, the field is zero. This kind of thinking is helpful in making qualitative descriptions of the problem, however some better means is needed to estimate p^2 as a function of depth. As has been done in the past, we suggest letting the intensity of rays passing through a depth be proportional to p^2.

Before we begin, let us make a few comments concerning the wave-ray or particle dualism of physics. Electrons are sometimes treated as waves and sometimes as particles. The formalism is chosen to be convenient to the problem. The same sort of dualism applies in classical physics to light and sound waves. If the apertures are very large and the wave fronts are nearly plane over many wave lengths, then it is both

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convenient and accurate to use the rays or normals to the wave fronts to calculate travel times and intensities. Wave descriptions are needed when the aperture is a small number of wave lengths. Also, wave descriptions are needed whenever rays are focused because one is looking at an interference phenomenon. Generally in these situations, one should say that a wave field is in the region of the receiver rather than that a ray or bundle of rays has passed through the receiver.

At large range, the wave front associated with a ray has spread enormously and thus has had many interactions with the upper and lower boundaries of the ocean. The surface and bottom limit the aperture of the transmission system and cause the energy to be spread over a range in depth. We will not attempt to calculate the spreading of the sound energy but will assert that it is reasonable to use the average ray density to estimate p^2.[1]

As shown on Fig. 3, the ocean at range r has been subdivided into small regions bounded by $r_1 \ldots r_m \ldots, r_M$ and $z_o, z_1, \ldots z_N$. The ray traces from the source are made at equal angle increments. A number of rays have been shown as passing through the regions. In keeping with our view that the wave field associated with the ray is rather large, we want to average the ray intensity over the distance Δr for the depth increments z_o, z_1, etc. By ray tracing, the number of rays passing through the area at $r_m, (z_{n+1} - z_n)$ is $P_{m,n}$. The average number of rays passing through the depth increment, $z_{n+1} - z_n$, is

$$\bar{P}_n \equiv \frac{1}{M} \sum_1^M P_{m,n} \qquad (3)$$

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[1] At the shorter ranges it is desirable to include the x-z spreading factor. M. Holl (1967).
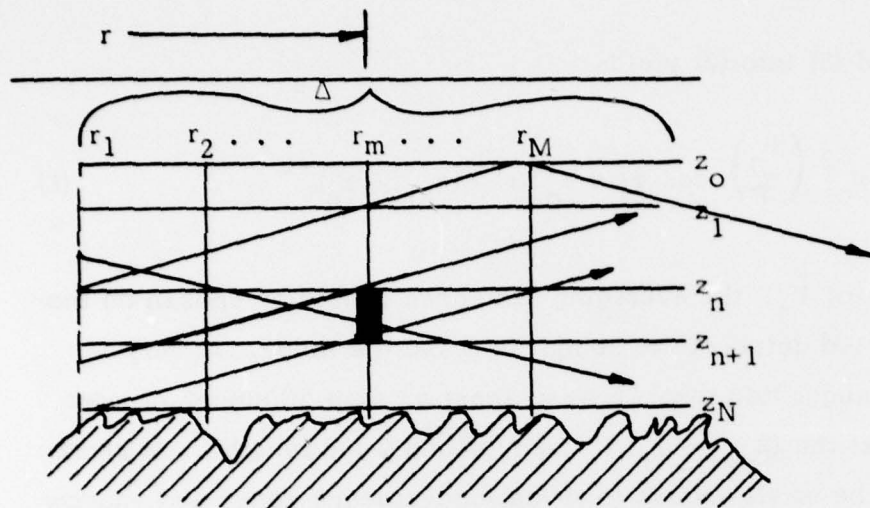
Fig. 3 Rays passing through small regions: The area at r_m, $(z_{n+1} - z_n)$
is darkened.

The total number of rays N is

$$\text{Total number of rays, } P \equiv \sum_{o}^{N} \overline{P}_n \tag{4}$$

(Note that, if no rays are lost in Δr, then P is the sum of the rays
passing through the wave duct.)

On the basis of our assumption that the mean square acoustical
pressure $\overline{p_n^2}$ at the depth $(z_n + z_{n+1})/2$ is proportional to $\overline{P}_n$ thus $\int \overline{p^2}\, dz$
and $\overline{p_n^2}$ are as follows:

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$$\overline{p_n^2} \, (z_{n+1} - z_n) = \overline{P}_n / P \int_0^h p^2 \, dz \qquad (5)$$

and substitution of (5) into (2) yields

$$\overline{p_n^2} \approx p_o^2 \left(\frac{\overline{P}_n}{P} \right) \cos \theta \Delta \theta \, r_o^2 \, r^{-1} (z_{n+1} - z_n)^{-1} \qquad (6)$$

In the computation of $\overline{P}_n$, the averaging distances should be chosen on the basis of the expected detail of the sound field and the range. At very large range, it is suggested that Δr be at least a "skip distance" or more than 60 km and that the $(z_{n+1} - z_n)$ be many sound wave lengths. At short range, Δr should be small enough to reveal convergence zones and shadow zones. Specification of analytical or quantitative values of Δr and $(z_{n+1} - z_n)$ is difficult and beyond the scope of this note.

Equation (6) can be used for any type of propagation. For example, the amount of energy trapped in the surface duct is given by $\Delta \theta_{sd}$ and its corresponding insonified depths. The effective sources for leakage signals can be approximated as being at the bottom of the duct and it is reasonable to assume the ray directions are slightly downward.

Deep water ray trace solutions often have rays that have had a few bottom interactions along the path. A way of treating these rays is suggested on Fig. 1b. Some of the energy is lost at each bottom reflection and part is reflected. At a reasonably large distance, the energy is again distributed between the maximum and minimum depths. Thus, we can include n bottom reflections as follows:

$$\int p_{br}^2 \, dz = p_o^2 \cos \theta_{br} \, \Delta \theta_{br} \, r_o^2 \, (r)^{-1} R_1^2 R_2^2 \ldots R_n^2 \qquad (7)$$

where $R_1, R_2, \ldots R_n$ are the average reflection coefficients along the

transmission path. Obviously, if the bottom is deep enough over part of the path such that the sound turns above the bottom, one only need include the losses where the sound reflects at the bottom. The total sound intensity is the sum of the contributions of refracted, surface and bottom reflected paths.

Although it is obviously a poor practice, sometimes one must assume a density distribution of the rays. If one assumes that the distribution is linear from the bottom to top turning points, then Eq. (6) can be written as follows:

$$\overline{p2}_n \, (z_{n+1} - z_n) \;\approx\; \overline{p^2} \, (z_{max} - z_{min})/N \tag{8}$$

and
$$\overline{P}_n \, N \;=\; P \tag{9}$$

$$\overline{p^2} \;\approx\; p_o^2 \, \cos \, \theta \, \Delta \, \theta \, r_o^2 \, (z_{max} - z_{min})^{-1} \, r^{-1} \tag{10}$$

But, we do not suggest the use of Eq. (10) if rough estimates of the density are available.

For the convenience of the reader, the transmission loss in dB is with the aid of Eq. (6) and including sound absorption, the following:

$$
\begin{aligned}
TL_n \;\approx\; & 10 \log r + Ar - 10 \log \, (\overline{P}_n/P) \\[6pt]
+ \;& 10 \log \, (z_{n+1} - z_n) - 10 \log \cos \theta \\[6pt]
- \;& 10 \log \Delta \theta - 10 \log r_o^2 , \, dB
\end{aligned} \tag{11}
$$

where

A   is the sound absorption in dB/m

$\overline{P}_n$ is the average number of rays passing through the depth increment $(z_{n+1} - z_n)$

P is the total number of rays at distance r

$\Delta \theta$ is the angular aperture of the source in radians

r, z_n, and r_o are in the same units, m.

Reference

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Acknowledgement